A Word by the Author

Russian mathematicians have suggested that ***mathematics is the gymnasium of the brains***. Likewise, in China, Mathematics Olympiad is ferociously practised by the large student population, all for a coveted place in one of the elite high schools.

Children who exhibit certain traits and penchant for numbers at the age of 5 or 6 years old, or even earlier, have great potential to be the mathematical olympians among their peers – provided they are groomed via a systematic, rigorous and routinized training.

Singapore was ranked 3rd in Mathematics in a recent TIMSS survey, after Hong Kong and Taiwan. Notably, China was not among the list of countries surveyed.

The most prestigious competition locally is **RIPMWC** (Raffles Institution Primary Mathematics World Competition). Meant for primary 6 students or younger, the top 50 to 60 or so participants are selected from Round 1 to compete in Round 2. Thereafter, 6 top participants emerge to take part in the world competition for primary school mathematics in Hong Kong. Another popular competition, also meant for primary 6 students, is **APMOPS** (Asia Pacific Mathematical Olympiad for Primary Schools), which is organized by Hwa Chong Institution since 1989. The following awards are being given at the end of two rounds of competition: ***Platinum, Gold, Silver and Bronze***.

At primary 5 level, the yearly **NMOS** (National Mathematical Olympiad of Singapore) competition has also captured the attention of parents since 2006, who eye NUS High as their most preferred high school.

The first series of books ***Maths Olympiad: Unleash the Maths Olympian in You!*** published in 2007 and 2008, has served as an ideal companion to students looking to establish a strong foundation in mathematics – be it for PSLE preparation or in hope that they might one day take part in the various local and international competitions. The books are, therefore, also first-choice materials for parents of primary 3 students looking for quality content in gifted programme training.

In this new edition you will find the following additions:

- The Pigeonhole Principle
- Values of Ones Digit
- The Shortest Path Method
- Defining New Operations

Enhancements have also been made to the following:

- Counting
- Speed
- Page Number Problem

The objective is to cater to increasingly smarter children who have been exposed to a wide variety of topics. Some of these topics, which overlap the local mathematics syllabus, have also been adopted by schools here for students to practice on.

I feel extremely privileged and honoured to be able to continue serving students in this field. My latest series ***Wicked Mathematics!*** is currently out on shelves.

For related courses and workshops, please visit **www.terrychew.org**.

Terry Chew
(2015)

Foreword

Occasionally, in some difficult musical compositions there are beautiful,
but easy parts - parts so simple a beginner could play them.
So it is with mathematics as well.
- Professor Sherman K. Stein -

Mathematical Olympiad has been widely practised in some countries due to the following characteristics:

- the wide range of topics that link mathematics to most everyday events,
- the witty and tricky nature of the problems that bring out the best in the students' thinking skills and creative imagination,
- encourages the students to use more than one method to solve the problems, thus stimulating them to think outside the box,
- the students will be equipped with abundant resources to devise their own methods in problem-solving due to the extensive training and exposure.

This book consolidates the materials that I have used to teach my students over the years. Although the problems are of Mathematical Olympiad type, I realised that all students can benefit by working on them. Built on and beyond the school syllabus, the importance of attitude and enthusiasm surpasses that of capability in learning Mathematical Olympiad.

Many children whom I have guided and their parents alike are mesmerized by the materials presented in this book.

I hope you and your child will be too!

Terry Chew
(2008)

CONTENTS

1 Looking for a Pattern

In mathematics, there are various patterns: some are relatively straightforward and others are more challenging. We, therefore, have to think outside the box and be flexible in our search for answers.

Besides adding or subtracting the terms in a number pattern, applying multiplication, division or even the use of any two arithmetic skills may help in the solving of the problems.

In **Fibonacci numbers**, the third term in the number pattern is the sum of the first and second terms; the fourth term is the sum of the second and third terms; the fifth term is the sum of the third and fourth terms, and so on. In essence, each term, after the first two terms, is the sum of the two preceding terms.

EXAMPLE

Complete each number pattern.

(a) 4, 7, 10, 13, ( ), ...

***Analysis*:** The difference between any two consecutive terms in the above number pattern is 3, so the next term must be 13 + 3 = **16**.

(b) 2, 6, 12, 20, ( ), ...

***Analysis*:** This is more interesting than the number pattern shown in **(a)**. The second term is 4 more than the first one. Thereafter, the difference between any two consecutive terms increases by 2.

$$2 + 4 = 6$$
$$6 + 4 + 2 = 12$$
$$12 + 4 + 2 + 2 = 20$$

The next term is, therefore, 20 + 4 + 2 + 2 + 2 = **30**.

(c) 2, 6, 18, ( ), ...

***Analysis*:** In the above number pattern, it is difficult to make sense of the difference between any two consecutive numbers. The difference between the first and second terms is 4. The difference between the second and third terms is 12. Observing the two differences will reveal that 12 is three times of 4. Hence the second term is three times the first term; the third term is three times the second term and so on.

$$6 \div 2 = 3$$
$$18 \div 6 = 3$$
$$18 \times 3 = 54$$

The next term is **54**.

(d) 44, 22, 20, 10, 8, ( ), ( )

***Analysis*:** The above number pattern uses two arithmetic skills: division and subtraction. The first term is divided by 2 and the second term is subtracted by 2.

$$44 \div 2 = 22 \qquad 22 - 2 = 20$$
$$20 \div 2 = 10 \qquad 10 - 2 = 8$$
$$8 \div 2 = 4 \qquad 4 - 2 = 2$$

The next two terms are **4** and **2** respectively.

(e) 1, 1, 2, 3, 5, 8, 13, ( ), ( ), 55, 89, ...

***Analysis*:** This number pattern is an example of Fibonacci numbers.

$$13 + 8 = 21$$
$$21 + 13 = 34$$

The two terms are **21** and **34** respectively.

PRACTICE

1 Complete each number pattern below.

(a) 1, 1, 2, 3, 5, ( ), ( ), ...

(b) 1, 1, 1, 1, 4, 7, 13, ( ), ...

(c) 3, 5, 9, 15, 23, 33, 45, ( ), ...

(d) 1, 3, 6, 10, ( ), 21, 28, 36, ...

(e) 0, 3, 8, 15, 24, ( ), 48, 63, ...

(f) 1, 2, 6, 24, 120, ( ), 5040, ...

(g) 0, 1, 3, 8, 21, 55, ( ), ( ), ...

(h) 1, 3, 7, 15, 31, ( ), 127, ...

(i) 1, 1, 3, 7, 13, ( ), 31, ...

(j) 1, 2, 5, 13, 34, 89, ( ), ( ), ...

2 *Ah, the magical trick of 9!*
Observe the pattern and write the correct answers in the brackets provided.

21 × 9 = 189

321 × 9 = 2 889

4 321 × 9 = 38 889

54 321 × 9 = ( )

654 321 × 9 = ( )

7 654 321 × 9 = ( )

87 654 321 × 9 = ( )

987 654 321 × 9 = ( )

3 Complete the Pascal Triangle and write the correct answers in the brackets provided.

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		( )		( )		( )		( )		1		
	1		( )		( )		( )		( )		( )		1	
1		( )		( )		( )		( )		( )		( )		1

4 *The marvel of multiplication of 101!*
Observe the pattern and write the correct answers in the brackets provided.

$101 \times 11 = 1111$

$101 \times 22 = 2222$

$101 \times 33 = 3333$

$101 \times 44 =$ ( )

$101 \times 55 =$ ( )

$101 \times 66 =$ ( )

$101 \times 77 =$ ( )

$101 \times 88 =$ ( )

$101 \times 99 =$ ( )