PREFACE

Pocket Notes Additional Mathematics is specially compiled to help students prepare for important tests and examinations.

Important formulae and concepts are presented in point form for quick recap and ease of understanding. Students will find this handy, practical and useful for test preparation.

The Editorial Team

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Algebra

Geometry and Trigonometry

Calculus

Surds

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$

Rationalizing of denominator:

- $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$(a + b)(a - b) = a^2 - b^2$

- $\frac{4}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{4(2 - \sqrt{3})}{2^2 - (\sqrt{3})^2} = \frac{8 - 4\sqrt{3}}{4 - 3} = 8 - 4\sqrt{3}$

Equations of Surds

Example

$\sqrt{x} = 3$

Squaring both sides,

$(\sqrt{x})^2 = 3^2$

$x = 9$

Laws of Indices

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^m \times b^m = (ab)^m$

$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

$a^0 = 1$

$a^{-p} = \left(\frac{1}{a^p}\right)$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Exponential Equations

Example

$2^x = 3$

Taking lg on both sides,

$\lg 2^x = \lg 3$

$x \lg 2 = \lg 3$

$x = \frac{\lg 3}{\lg 2}$

≈ 1.58

When the unknown is at the power position, take lg or ln on both sides of the equation.

Laws of Logarithms

$\log_a (xy) = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$*\log_a (x^r) = r \log_a x$

$\log_a b = \frac{\log_c b}{\log_c a}$

(Change base to c)

$\log_a a = 1$

$\log_a 1 = 0$

When using logarithm laws, the logarithms must only have an invisible '1' in front, or use * to bring number up.

Logarithmic Equations

1. *1 Logarithmic Function*

Example

Solve the equation $\log_9(7 + 3x) = 2$.

Solution:

$\log_9 (7 + 3x) = 2$

$7 + 3x = 9^2$

$3x = 81 - 7$

$x = 24\frac{2}{3}$

2. *Logarithmic Functions with Same Base*

Example

Solve the equation $3 + \log_2(x + 1) = 2\log_2(x - 5)$.

Solution:

$$3 + \log_2 (x + 1) = 2\log_2 (x - 5)$$
$$3 = 2\log_2(x-5) - \log_2(x+1)$$
$$2\log_2(x - 5) - \log_2(x + 1) = 3$$
$$\log_2(x - 5)^2 - \log_2(x + 1) = 3$$
$$\log_2 \frac{(x - 5)^2}{(x + 1)} = 3$$
$$\frac{(x - 5)^2}{(x + 1)} = 2^3$$
$$(x - 5)^2 = 8(x + 1)$$
$$x^2 - 10x + 25 = 8x + 8$$
$$x^2 - 18x + 17 = 0$$
$$(x - 17)(x - 1) = 0$$
$$x = 17 \text{ or } x = 1 \text{ (rej.)}$$

Check validity:
For $\log_a x \rightarrow a > 0$,
$a \neq 1$,
$x > 0$
For $\ln x \rightarrow x > 0$

3. *Logarithmic Functions with Different Bases*

Example

Solve the equation $\log_{27} x^3 + \log_3 (x - 2) = 1$.

Solution:

$$\log_{27} x^3 + \log_3 (x - 2) = 1$$

$$\frac{\log_3 x^3}{\log_3 27} + \log_3 (x - 2) = 1$$

$$\frac{\log_3 x^3}{\log_3 3^3} + \log_3 (x - 2) = 1$$

$$\frac{3\log_3 x}{3} + \log_3 (x - 2) = 1$$

$$\log_3 x + \log_3 (x - 2) = 1$$

$$\log_3 [x\,(x - 2)] = 1$$

$$x(x - 2) = 3^1$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1 \text{(rej.)}$$

Check validity:
For $\log_a x \rightarrow a > 0$,
$a \neq 1$,
$x > 0$
For $\ln x \rightarrow x > 0$

4. *Repeated Logarithmic Functions*

Example

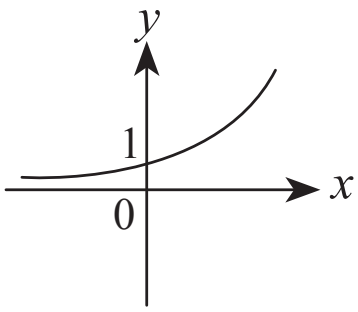
Solve the equation $\log_3 x^3 = (\log_3 x)^2$.

Solution:

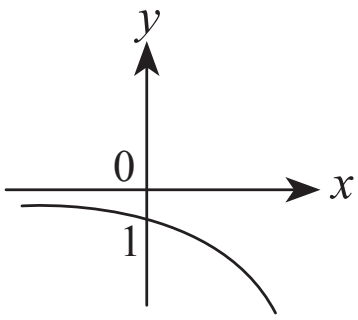
$$
\begin{aligned}
3\log_3 x &= (\log_3 x)^2 \\
\text{Let } y &= \log_3 x \\
3y &= (y)^2 \\
y^2 - 3y &= 0 \\
y(y-3) &= 0 \\
y &= 0 \quad \text{or} \quad y = 3 \\
\log_3 x &= 0 \quad \text{or} \quad \log_3 x = 3 \\
x &= 3^0 \quad \text{or} \quad x = 3^3 \\
&= 1 \qquad\qquad = 27
\end{aligned}
$$

Graphs of Exponential and Logarithmic Functions

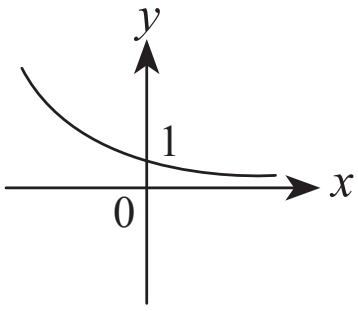
$y = e^x$



$y = -e^x$



$y = e^{-x}$



$y = \ln x$