

PREFACE

Pocket Notes Additional Mathematics is specially compiled to help students prepare for important tests and examinations.

Important formulae and concepts are presented in point form for quick recap and ease of understanding. Students will find this handy, practical and useful for test preparation.

The Editorial Team

CONTENTS

Algebra

Surds	1
Equations of Surds	1
Laws of Indices	2
Exponential Equations	2
Laws of Logarithms	3
Logarithmic Equations.....	3
Graphs of Exponential and Logarithmic Functions..	7
Nature of Roots of a Quadratic Equation.....	8
Intersection of a Curve and a Line	9
Sum and Product of Roots	10
Quadratic Inequalities	11
Factor and Remainder Theorem.....	12
Cubic Equations	12
Partial Fractions	13
Modulus Functions.....	14
Graphs of Modulus Functions.....	15
Binomial Theorem	16

Geometry and Trigonometry

Coordinate Geometry.....	17
Properties of Special Quadrilaterals.....	18
Linear Law	20

Curves	21
Circles	22
Trigonometric Ratios of General Angles	23
Trigonometric Ratios of Special Angles	24
Graphs of Trigonometric Functions	25
Trigonometric Identities and Formulae.....	27
Trigonometric Equations.....	29
Proofs in Plane Geometry	31

Calculus

Rules of Differentiation	33
Equations of Tangent and Normal	34
Rate of Change.....	34
Increasing and Decreasing Functions	34
Maxima and Minima Problems.....	35
Differentiation of Exponential Functions	36
Differentiation of Logarithmic Functions	36
Differentiation of Trigonometric Functions	37
Rules of Integration.....	38
Equation of a Curve	38
Integration of Trigonometric Functions	39
Integration of Exponential Functions.....	39
Integration of Reciprocal Functions.....	39
Definite Integrals.....	40
Area under a Curve	40
Kinematics	42

Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$$

Rationalizing of denominator:

$$\bullet \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$(a + b)(a - b) = a^2 - b^2$$

$$\bullet \frac{4}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{4(2 - \sqrt{3})}{2^2 - (\sqrt{3})^2} = \frac{8 - 4\sqrt{3}}{4 - 3} = 8 - 4\sqrt{3}$$

Equations of Surds

Example

$$\sqrt{x} = 3$$

Squaring both sides,

$$(\sqrt{x})^2 = 3^2$$

$$x = 9$$

Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^m \times b^m = (ab)^m$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$a^0 = 1$$

$$a^{-p} = \left(\frac{1}{a^p}\right)$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Exponential Equations

Example



$$2^x = 3$$

Taking lg on both sides,

$$\lg 2^x = \lg 3$$

$$x \lg 2 = \lg 3$$

$$x = \frac{\lg 3}{\lg 2}$$

$$\approx 1.58$$

When the unknown is at the power position, take lg or ln on both sides of the equation.

Laws of Logarithms

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$*\log_a(x^r) = r \log_a x$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

(Change base to c)

$$\log_a a = 1$$

$$\log_a 1 = 0$$

When using logarithm laws, the logarithms must only have an invisible '1' in front, or use * to bring number up.

Logarithmic Equations

1. 1 Logarithmic Function

Example

Solve the equation $\log_9(7 + 3x) = 2$.

Solution:

$$\log_9(7 + 3x) = 2$$

$$7 + 3x = 9^2$$

$$3x = 81 - 7$$

$$x = 24\frac{2}{3}$$

2. Logarithmic Functions with Same Base

Example



Solve the equation $3 + \log_2(x + 1) = 2\log_2(x - 5)$.

Solution:

$$3 + \log_2(x + 1) = 2\log_2(x - 5)$$

$$3 = 2\log_2(x - 5) - \log_2(x + 1)$$

$$2\log_2(x - 5) - \log_2(x + 1) = 3$$

$$\log_2(x - 5)^2 - \log_2(x + 1) = 3$$

$$\log_2 \frac{(x - 5)^2}{(x + 1)} = 3$$

$$\frac{(x - 5)^2}{(x + 1)} = 2^3$$

$$(x - 5)^2 = 8(x + 1)$$

$$x^2 - 10x + 25 = 8x + 8$$

$$x^2 - 18x + 17 = 0$$

$$(x - 17)(x - 1) = 0$$

$$x = 17 \text{ or } x = 1 \text{ (rej.)}$$

Check validity:

For $\log_a x \rightarrow a > 0$,

$a \neq 1$,

$x > 0$

For $\ln x \rightarrow x > 0$

3. Logarithmic Functions with Different Bases

Example



Solve the equation $\log_{27} x^3 + \log_3(x - 2) = 1$.

Solution:

$$\log_{27} x^3 + \log_3(x - 2) = 1$$

$$\frac{\log_3 x^3}{\log_3 27} + \log_3(x - 2) = 1$$

$$\frac{\log_3 x^3}{\log_3 3^3} + \log_3(x - 2) = 1$$

$$\frac{3\log_3 x}{3} + \log_3(x - 2) = 1$$

$$\log_3 x + \log_3(x - 2) = 1$$

$$\log_3 [x(x - 2)] = 1$$

$$x(x - 2) = 3^1$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1(\text{rej.})$$

Check validity:

For $\log_a x \rightarrow a > 0$,

$a \neq 1$,

$x > 0$

For $\ln x \rightarrow x > 0$

4. Repeated Logarithmic Functions

Example



Solve the equation $\log_3 x^3 = (\log_3 x)^2$.

Solution:

$$3\log_3 x = (\log_3 x)^2$$

$$\text{Let } y = \log_3 x$$

$$3y = (y)^2$$

$$y^2 - 3y = 0$$

$$y(y - 3) = 0$$

$$y = 0 \quad \text{or} \quad y = 3$$

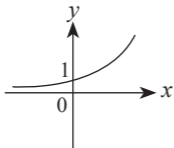
$$\log_3 x = 0 \quad \text{or} \quad \log_3 x = 3$$

$$x = 3^0 \quad \text{or} \quad x = 3^3$$

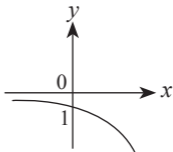
$$= 1 \qquad \qquad = 27$$

Graphs of Exponential and Logarithmic Functions

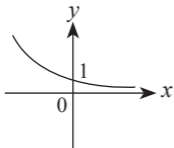
$$y = e^x$$



$$y = -e^x$$



$$y = e^{-x}$$



$$y = \ln x$$

